O K L A H O M A   S T A T E   U N I V E R S I T Y

S C H O O L   O F   E L E C T R I C A L   A N D   C O M P U T E R   E N G I N E E R I N G

ECEN 5713 System Theory
Spring 1997
Final Exam

Name : ______________________________

Student ID: ________________________________

E-Mail Address:_______________________________________

NAME:______________________

<u>Problem 1</u> (Realization)

Find a minimal "observable" canonical form realization (i.e., its simulation diagram and state space representation) for the following MISO system described by

$$H(s) = \left[\frac{2s+3}{s^3+4s^2+5s+2} \quad \frac{s^2+2s+2}{s^4+3s^3+3s^2+s} \right].$$

NAME:______________________

<u>Problem 2</u> (Solution of Dynamic Equation)

Given

$$x(k+1) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 3 \end{bmatrix} u(k) .$$

$$y(k) = \begin{bmatrix} 1 & 3 \end{bmatrix} x(k)$$

For zero input (i.e., $u(k)$=0), $y(0) = 1$ and $y(1) = 1$. Determine its initial condition, $x(0)$.

NAME:______________________

<u>Problem 3</u> (Adjoint System)

Consider

$$\dot{x} = A(t)x + B(t)u$$
$$y = C(t)x$$

and its adjoint equation

$$\dot{z} = -A^T(t)z + C^T(t)v$$
$$w = B^T(t)z \quad .$$

Let $G(t,\tau)$ and $G_a(t,\tau)$ be their impulse response matrices. Show that

$$G(t,\tau) = G_a^T(\tau,t) \; .$$

If A, B, and C are constant (independent of time) matrices, then show

$$H(s) = -H_a^T(-s) \; ,$$

where $H(s)$ and $H_a(s)$ are their transfer function matrices.

NAME:______________________

<u>Problem 4</u> (Equivalent Transformation)

Consider discrete-time system representations

$$x(k+1) = A(k)x(k) + B(k)u(k)$$
$$y(k) = C(k)x(k) + D(k)u(k)$$

and

$$\bar{x}(k+1) = \bar{A}(k)\bar{x}(k) + \bar{B}(k)u(k)$$
$$y(k) = \bar{C}(k)\bar{x}(k) + \bar{D}(k)u(k)$$

where $\bar{x}(k) = P(k)x(k)$ with $|P(k)| \neq 0, \forall\, k \geq k_0$. Determine the relations between A, B, C, D and $\bar{A}, \bar{B}, \bar{C}, \bar{D}$ for two representations to be equivalent. What would be the relation between state transition matrices $\Phi_A(k, k_0)$ and $\Phi_{\bar{A}}(k, k_0)$? And what would be the relation between impulse response matrices $G_A(k, l)$ and $G_{\bar{A}}(k, l)$?

NAME:______________________

<u>Problem 5</u> (Controllable and Observable Reduction)
Reduce the following dynamic equation

$$\dot{x} = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} x$$

to a controllable *<u>and</u>* observable system.